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Eurocode 2: Span/depth ratios for RC slabs and beams

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Synopsis

Eurocode 2 introduces new span/depth rules for the design of reinforced concrete beams and slabs. These are investigated from practical and theoretical points of view and serious problems found. The present UK National Annex imposes requirements which are impossible to comply with, making EC 2 unusable in the UK until it is revised. The proposed EC 2 recommendations offer a choice between an over-conservative simple table, or a formula for calculating allowable ratios which is impractical to use and contains serious errors. In particular its wording is ambiguous, which makes the effects of varying steel stress unclear, and it exaggerates the effect that increasing concrete strength has on deflection. Until these problems are sorted out, it is recommended that present BS 8110 limits are retained but it is recommended that an error in these should be corrected to improve economy.

Introduction

In reinforced concrete design, deflection is normally controlled by limiting the span/depth ratio of a beam or slab. This paper considers the span/depth rules proposed by Eurocode 2 [1] and compares them with existing UK practice.

To make the discussion easier for UK engineers to follow, the normal UK terms 'loads' and 'f_{cu}' are used rather than the Eurocode terminology of 'actions' and $f_{ck,cube}$. As deflection is a serviceability condition, analysis has been based on service loads and these are generally taken as the design characteristic loads. In Eurocode 0, characteristic loads can be factored down for serviceability calculations. For clarity, these reduction factors are not used in the main analysis, so that technical aspects of the design rules can be compared on a 'like for like' basis. The effects of the EC 0 proposed reduction factors on service loads are then discussed separately.

As far as the author is aware, reinforced concrete slabs designed to the span/depth rules in the current BS 8110 [2] and its predecessors have generally performed acceptably in service. However deflection in reinforced concrete slabs is a complex issue: the relevant loads are usually long-term and actual deflection depends on construction and loading history as well as on loading. A full analysis of the relevant test data and theory to try establish exactly what the 'correct' span/depth ratios are for all situations would be a major research project, outside the scope of the present paper. The analysis in the paper is based on comparison between results from the proposed Eurocode 2 rules, results from existing codes and basic theoretical considerations.

Span/depth limits play a critical role in design: they commonly determine floor thicknesses and beam depths. These in turn dictate the weight of the structure, headroom and storey heights and can have a major effect on the cost of a building. Slab thicknesses and beam depths are generally decided early in the development of the design and are difficult to change later, so design rules are needed which give sensible, consistent results and can be applied early in the design process.

In the past, this was easy: CP 114 [3] gave a simple table of span/depth ratios for beams and slabs which could be applied directly, without any calculations. The recommendations were rather crude but in most cases, as far as the author is aware, they produced serviceable, reasonably economical structures.

In 1972, CP110 [4] introduced new recommendations based on research by Beeby [5]. In these, the allowable span/depth ratio varied depending on the steel tensile stress and the amount of tensile and compressive reinforcement. The new rules were an improvement from a theoretical point of view but unfortunately, because of the way they were presented, the engineer could only check whether the span/depth ratio was acceptable after the reinforcement design had been completed. For initial scheme design, engineers had to estimate slab thicknesses by guesswork and they tended to make conservative assumptions in order to avoid problems later. In theory, slab thicknesses could have been reduced later where appropriate, once full calculations had been prepared but in practice this was rarely done. As a result, despite the theoretical advantages of the CP 110 span/depth rules, they gained a reputation for producing overweight, uneconomical designs compared with CP 114.

The problem was solved by changing the presentation of the CP 110 rules: instead of relating the span/depth factors to reinforcement area and stress, they were presented in terms of M/bd^2, which allowed them to be checked earlier in the design calculation [6] and CP110's successor, BS 8110, adopted this approach. The derivation of the BS 8110 recommendations is explained in the Handbook to British Standard BS 8110:1985 [7]. Based on this work, simple tables of allowable span/depth ratios for slabs were also published which combined the accuracy of CP110 with the simplicity of CP 114 [8] and these were included in the ICE/IStructE 'Green Book' for limit state design [9] and the IStructE 'Gold Book' for permissible stress design of reinforced concrete building structures [10].

Eurocode 2

Eurocode 2 introduces a new method for calculating allowable span/depth ratios for reinforced concrete beams and slabs.

Cl. 7.4.2 Table 7.4N gives span/depth limits for beams and slabs with a service tensile stress f_s of 310N/mm². At first sight, this looks refreshingly simple: for a simply supported beam or slab with 0.5% tensile reinforcement the L/d limit is 20; with 1.5% reinforcement the limit is 14 and corresponding limits are specified for continuous beams, flat slabs and cantilevers. However the apparent simplicity of Table 7.4N comes at a price, as its limits are conservative for lightly-loaded slabs. It also shares the weakness of the old CP 110 rules: for initial sizing of slabs and beams the engineer has to rely on conservative guesswork, as the allowable span/depth ratio cannot be checked until the reinforcement design is complete.

Table 1 shows slab thicknesses required to support an imposed load of 5kN/m² over a span of 4.5m according to EC 2 Table 7.4N. These are compared with the thickness required by Table 3 in the IStructE 'Green Book' (which gives approximate span/depth ratios for BS 8110 designs, based on superimposed load) and Table 6c in the 'Gold Book', (which gives exact ratios based on total load). As the two UK documents (which are both based on BS 8110) use different design tensile stresses from EC 2, a comparison is also given of slab thicknesses required by BS 8110 for design to the EC 2 standard steel service stress f_s = 310N/mm^2. (Reinforcement bars assumed H12 with 20mm cover in all cases.)

Table 1 Slab thickness for 4.5m span

	Simply supported		Continuous	
	L/d	slab thickness	L/d	slab thickness
IStructE/ICE 'Green Book'	23	222	30	176mm
IStructE 'Gold Book'	27	193	37	148mm
BS 8110 f_s = 310N/mm²	24.6	210	34.3	158mm
EC 2 Table 7.4N	20	251	26	199mm

The 'Green Book' limits are based on f_s = 333N/mm², as per the current edition of BS 8110 (which is over-conservative - see later); the Gold Book figures are based on f_s = 275N/mm² and the EC 2 figures are based on f_s = 310N/mm². As can be seen, slabs designed to EC 2 Table 7.4N would be 23-29mm thicker than similar designs to the Green Book, 51-58mm thicker than Gold Book designs and 41 mm thicker than BS 8110 designs based on f_s = 310N/mm². Thus using EC 2 Table 7.4N would increase the weight of the structure and also storey heights, substantially increasing the cost of the building.

As an alternative to Table 7.4N, EC 2 allows span/effective depth limits (L/d) to be calculated from equations 7.16a, 7.16b and 7.17:

$L/d = K(11+(1.5\sqrt{f_{ck}} \times \rho_0/\rho) + 3.2\sqrt{f_{ck}}(\rho_0/\rho - 1)^{1.5})$ if $\rho_0 \leq \rho$ (7.16a)

$L/d = K(11+(1.5\sqrt{f_{ck}} \times \rho_0/(\rho - \rho')) + \sqrt{f_{ck}}\ \sqrt{(\rho_0'/\rho)})/12)$ if $\rho_0 > \rho$ (7.16b)

where K is obtained from Table 7.4N, with values 1.0 (simply supported), 1.5 (continuous), 1.3 (continuous - end span), 1.2 (flat slab), 0.4 (cantilever).

ρ_0 = reference reinforcement ratio = $10^{-3}\sqrt{f_{ck}}$

ρ = required tension reinforcement ratio

ρ' = required compression reinforcement ratio

f_{ck} = concrete cylinder strength (N/mm²)

Cl. 7.4.2 then goes on to state: 'Expressions (7.16a) and (7.16b) have been derived on the assumption that the steel stress ... at SLS at a cracked section at the midspan of a beam or slab or at the support of a cantilever is 310MPa (corresponding roughly to f_{yk} = 500MPa).

Where other stress levels are used, the values obtained using Expression (7.16) should be multiplied by $310/\sigma_s$. It will normally be conservative to assume that:

$310/\sigma_s = 500/(f_{yk}\ A_{sreq}/A_{sprov})$ (7.17) where

σ_s = tensile stress at midspan (support for cantilevers) under the design load at SLS

A_{sprov} = area of steel provided at this section

A_{sreq} = area of steel required at this section for ultimate limit state.'

(The calculated L/d is reduced by $7/L_{eff}$ for beams and slabs spanning more than 7m and $8.5/L_{eff}$ for flat slabs spanning more than 8.5m.)

From a practical point of view, these recommendations are even worse than the old CP 110 rules: not only must the reinforcement design be completed before the span/depth ratio can be calculated but there are no tabulated values to streamline the process. However the engineer who wishes to produce an economical design has no alternative but to try to get to grips with equations 7.16a, 7.16b and 7.17.

Comparison with UK practice

Before comparing EC 2 with UK practice, it is necessary first of all to deal with an error in the current edition of BS 8110. In CP 110, the service stress associated with 460N/mm² reinforcement was f_s= 0.58f_y = 267N/mm²; this corresponded to a materials factor of f_y = 1.15 and an average load factor of (1.4+1.6)/2 = 1.5. In the first edition of BS 8110 this was increased to f_s = 0.625f_y = 288N/mm² (average load factor 1.39). Then in 2002 an amendment reduced the material factor to 1.05 and f_s was increased to 0.667f_y = 307N/mm².

In 2005, following the increase in high tensile steel f_y to 500N/mm², BS 8110 was again amended: the material factor reverted to 1.15 but unfortunately the formula for f_s in Table 3.10 was left unchanged at 0.667f_y increasing f_s to 333N/mm². This was clearly a mistake, as it would correspond to an average load factor of only 1.3, which is less than the minimum possible for a beam or slab supporting dead load and imposed loads. Restoring the service stress to its 1985 value of 0.625f_y (load factor 1.39) would reduce it to f_s = 0.625 x 500 = 312N/mm² but even this is unnecessarily conservative: the realistic minimum average load factor is about 1.45, which would give f_s = 0.6f_y = 300N/mm². Correcting this error in the current BS 8110 would significantly reduce slab thicknesses and improve economy.

Table 2 Comparison of L/d limits for simply supported slab design (f_s = 310N/mm²)

Total Service Loading (DL+LL)	5kN/m²	10kN/m²	20kN/m²
BS 8110	28	24.6	21.5
EC 2 7.4N (0.5% steel)	20	20	20
EC 2 Eq. 7.16 (f_{cu} = 30N/mm²)	36.1	27.9	21.8

Table 2 compares the allowable span/effective depth ratios for slabs designed to BS 8110 and EC 2 based on f_{cu} = 30N/mm² and the EC 2 standard steel stress f_s = 310N/mm².

As can be seen, EC 2 Table 7.4N is very conservative in most cases. For slabs supporting very heavy loading, EC 2 Eq. 7.16 and BS 811 0 give very similar results but for light and medium loadings, EC 2 Eq. 7.16 allows substantially thinner slabs than BS 8110.

Effect of concrete strength

In EC 2 Eq. 7.16, increasing the concrete strength increases the allowable span/effective depth ratio. Table 3 shows the results for f_{cu} = 30N/mm² and 50N/mm² (f_s = 310N/mm²).

As can be seen, in EC 2, increasing f_{cu} from 30N/mm² to 50N/mm² increases the allowable *L*/*d* by 14%. When this is combined with other factors, the result is that a lightly loaded slab made with high strength concrete and designed to EC 2 could have an effective depth less than 70% of a comparable slab designed to BS 8110.

Table 3 Variation of L/d limits with concrete strength (simply supported slab)

Total Service Loading (DL+LL)	5kN/m²	10kN/m²	20kN/m²
BS 8110	28	24.6	21.5
EC 2 Eq. 7.16 (f_{cu} = 30N/mm²)	36.1	27.9	21.8
EC 2 Eq. 7.16 (f_{cu} = 50N/mm²)	41.1	31.7	24.7

Effect of reinforcement stress

In BS 8110, the allowable span/effective depth ratio may be increased by reducing the tensile stress in the reinforcement. (Compression reinforcement may also be used but this is less common and not considered in the present analysis.) This is often done where one span of a slab is 'over the limit' and it has been claimed that in some cases maximum economy is obtained by reducing the steel service stress to as low as 200N/mm², to minimise slab thickness [11].

In EC 2, if f_s varies from 310N/mm², Eq. 7.17 is used to adjust the allowable *L/d.* However the reinforcement ratio ρ also appears in equations 7.16a and 7.16b and EC 2 does not make clear how this should be calculated when f_s varies from 310N/mm². The most obvious interpretation would be to assume that ρ is the actual tensile reinforcement ratio (i.e. A_{sprov}/bd). Table 4 shows the results from EC 2 if it is interpreted in this way for a simply supported slab with a total load of 10kN/m² and these are compared with BS 8110.

Table 4 Variation of EC 2 L/d limits with steel service stress (simply supported slab, (f_{cu} = 30N/mm²), total service loading 10kN/m²)

f_s (N/mm²	150	200	250	310
BS 8110	31.1	29.4	27.4	24.6
EC 2 Eq. 7.16 Interpretation A	31.5	28.1	27.6	27.9
EC 2 Eq. 7.16 Interpretation B	36.1	31.7	29.6	27.9

As can be seen, in BS 8110 the allowable *L/d* increases steadily with reducing tensile stress: reducing f_s from 310N/mm² to 200N/mm² increases the allowable *L/d* by 20%. On the other hand in EC 2, if 'Interpretation A' is applied, reducing f_s from 310N/mm² to 200N/mm² makes almost no difference at all to the allowable *L/d.*

However it is possible to interpret this part of EC 2 in a different way: if Eq. 7.16 was derived on the assumption of a steel stress of 310N/mm² then, rather than being the actual reinforcement ratio (A_{sprov}/bd), could 'ρ' be *the reinforcement ratio which would have been required if the steel service stress had been 310N/mm²,* i.e. (A_{sprov}/bd) x (σ_s/310)? The allowable *L/d* ratios for various steel tensile stresses based on this interpretation for a simply supported slab with a total load of 10kN/m² are shown in Table 4.

As can be seen, if 'Interpretation B' of EC 2 is correct, the allowable *L/d* would increase in a more believable way as steel stress is reduced. However, whereas in Interpretation 'A' the effect was much less than in BS 8110, in Interpretation B it is greater than in BS 8110: reducing f_s from 310N/mm² to 150N/mm² increases the allowable *L/d* by 26% in BS 8110 but in EC 2 it increases by 29%.

Table 5 shows how the modification factor on allowable span/effective depth ratio varies with changing steel tensile stress for a beam or slab, compared with a basic *L/d* ratio of 20. The factors are calculated for a section with a concrete cube strength f_{cu} = 30N/mm² and M_{SLS}/bd^2 = 1.

Table 5 Variation of allowable L/d modification factor with steel service stress (M_{SLS}/bd^2 = 1, f_{cu} = 30N/mm², basic L/d ratio 20)

f_s (N/mm²	150	200	250	310
BS 8110	1.69	1.51	1.34	1.13
EC 2 Eq. 7.16 Interpretation A	1.69	1.4	1.3	1.35
EC 2 Eq. 7.16 Interpretation B	2.79	2.09	1.67	1.35

As can be seen, if 'Interpretation A' is adopted, the EC 2 figures follow a very peculiar trend: the factor is almost constant between 310N/mm² and 200N/mm² but then it rises quite sharply once f_s drops below 200N/mm². EC 2 interpretation 'B' produces a more believable general trend but the rise in allowable *L/d* as f_s reduces is very rapid:

in EC 2 halving the steel stress increases the allowable L/d by 100%, whereas in BS 8110 the corresponding increase is less than 50%.

EC 2 Cl. 7.4.2(2) increases the allowable L/d by the ratio $310/\sigma_s$, where σ_s is the steel service stress, so doubling the reinforcement (i.e. halving the steel service stress) doubles the allowable span/depth ratio. This relationship would be true for a steel beam. However in a cracked section reinforced concrete beam increasing the reinforcement area not only reduces the steel tensile stress but it also shifts the neutral axis. Therefore in a cracked reinforced concrete section, the reduction in deflection will be less than the reduction in steel stress. In an uncracked section, the steel stress will have even less effect on deflection.

Therefore EC 2 Cl. 7.4.2(2) is clearly incorrect and overestimates the effect that reducing steel stress has on beam deflection, particularly where concrete tension zone stiffening has been included in the analysis.

Allowance for concrete tension zone stiffening

Effects related to concrete tension zone stiffening are: (i) it increases stiffness when concrete stresses are low (low M/bd^2) and (ii) when concrete strength is high, this increases lever arm and it also increases concrete tensile strength, so the effect of tension zone stiffening effect is greater.

For a simply-supported beam with f_{cu} = 30N/mm², f_s = 310N/mm² and a service moment intensity of M_{SLS}/bd^2 = 1.21 (i.e. a moderately loaded beam or a heavily loaded slab), BS 8110 and EC 2 both recommend the same L/d limit: 21.3. Table 6 shows how the multipliers on this basic span/depth ratio of 21.3 vary for different load intensities (expressed as M_{SLS}/bd^2). Table 7 shows the variation for different concrete strength relative to f_{cu} = 30N/mm² (M/bd^2 = 1.21, f_s = 310N/mm²).

Table 6 Variation of modification factor on L/d with M_{SLS}/bd^2 (f_{cu} = 30N/mm², f_s = 310N/mm², basic L/d ratio 21.3)

M/bd^2	0.3	0.5	1	1.21	2	3
BS 8110	1.48	1.31	1.06	1	0.85	0.76
EC 2	8.04	3.59	1.27	1	0.76	.66

Table 7 Variation of allowable L/d modification factor with f_{cu} relative to value at f_{cu} = 30N/mm², f_s = 310N/mm²)

		f_{cu} (N/mm²)	20	30	40	60
BS 8110	M/bd^2	0.5	1	1	1	1
		1	1	1	1	1
		2	1	1	1	1
EC 2	M/bd^2	0.5	0.59	1	1.36	2.39
		1	0.67	1	1.37	2.25
		2	0.86	1	1.14	1.48

As can be seen from Tables 6 & 7, although BS 8110 and EC 2 give very similar results when f_{cu} = 30N/mm² and M_{SLS}/bd^2 of 1-1.5, away from these conditions the differences are quite remarkable:

(i) reducing M_{SLS}/bd^2 from the reference value of 1.21 to 0.3 increases the L/d multiplier in BS 8110 from 1.0 to 1.48 but in EC2 it increases to an astonishing 8.04;

(ii) increasing concrete strength does not affect allowable L/d in BS 8110 but in EC 2, for M_{SLS}/bd^2 up to 1.0, allowable L/d is roughly proportional to concrete strength: reducing f_{cu} from 30N/mm² to 20N/mm² reduces allowable L/d factor to 59-67% and increasing it to 60N/mm² more than doubles it.

The L/d ratios permitted by BS 8110 and EC 2 can be compared with what would be expected from simple analysis of cracked and uncracked reinforced concrete sections. If the allowable deflection is $L/250$, it can be shown that

allowable $L/d = (24E/625) \times (1/bd^3)/(M_{SLS}/bd^2)$,

where E is Young's Modulus.

If I is in concrete units, m *is* the modular ratio and

E = 200kN/mm², then

$L/d = (7680/m) \times (1/bd^3)/(M_{SLS}/bd^2)$

Based on the tabulated concrete properties and creep factors in EC 2 for long term loading, $m = 21$ for f_{cu} = 30N/mm² and $m = 13$ for f_{cu} = 60N/mm². Figure 1 shows calculated L/d limits for f_{cu} = 30N/mm² (simply supported beam) for cracked and uncracked sections and compares these with BS 8110 and EC 2 limits; Fig. 2 shows the corresponding figures for f_{cu} = 60N/mm².

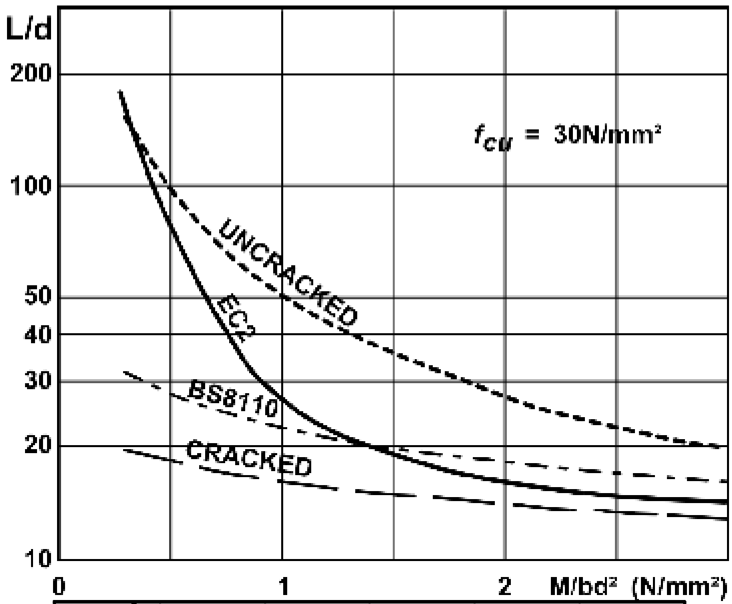

Fig. 1 Calculated L/d limits for f_{cu} = 30N/mm² (simply supported beam) for cracked and uncracked sections compared with the BS 8110 and EC 2 limits

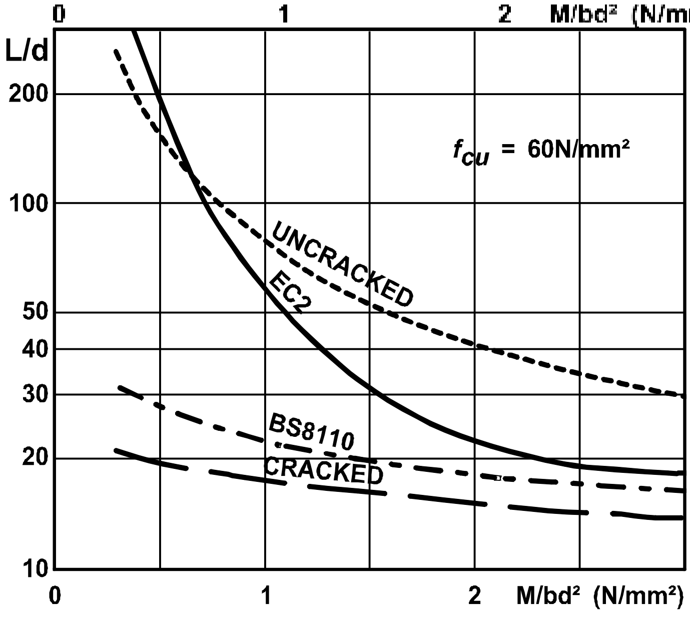

Fig. 2 Calculated L/d limits for f_{cu} = 60N/mm² (simply supported beam) for cracked and uncracked sections compared with the BS 8110 and EC 2 limits

As can be seen, EC 2 is more conservative than BS 8110 at high values of M_{SLS}/bd^2 but at low M_{SLS}/bd^2 EC 2 gives much higher L/d ratios. When $M_{SLS}/bd^2 < 0.5$ (f_{cu} = 30N/mm²), or $M_{SLS}/bd^2 < 1$ (f_{cu} = 60N/mm²), EC 2 Eq. 7.16 gives results which approximate to the theoretical results for an uncracked concrete section.

Beeby, Scott and Jones have reviewed tension zone stiffening effects in concrete, following recent tests at Leeds and Durham Universities [12]. They found that long term tensile strength was much lower than the short term value and that 'the rate of decay of tension stiffening is much more rapid than has previously been assumed'. They recommended that in BS 8110 theoretical deflection calculations the assumed concrete tension at the level of the reinforcement should be limited to 0.55N/mm². When the more exact 'ICE Technical Note 372' method for calculating deflection is used they recommended that the concrete tensile stress at the outer face of the concrete should be limited to a maximum of $0.55f_t$, where f_t is the concrete tensile strength.

Table 8 shows the calculated uncracked section concrete tensile stress at M_{SLS}/bd^2 = 0.5 (f_{cu} = 30N/mm²) and 1.0 (f_{cu} = 60N/mm²) and compares this with the tensile stress limits recommended by Beeby, Scott &James. The limiting stress for ICE Note 372 is taken as $0.55f_{ctm}$, where f_{ctm} is the mean concrete tensile strength from EC 2.

Table 8 Uncracked section: concrete tensile stress compared with revised BS 8110 and ICE Technical Note 372 (ref. Beeby, Scott & Jones)

	f_{cu} = 30N/mm², M_{SLS}/bd^2 = 0.5	f_{cu} = 60N/mm², M_{SLS}/bd^2 = 1
calculated conc. tension at steel	1.55N/mm²	3.05N/mm²
rev. BS 8110 limit	0.55N/mm²	0.55N/mm²
calculated conc. tension, bottom face	2.11N/mm²	4.16N/mm²
rev. ICE note 372 limit	1.60N/mm²	2.42N/mm²

As can be seen, in all cases the tensile stress in the uncracked section would exceed the recommended limits. For f_{cu} = 30N/mm², it is 32% greater than the value recommended by Beeby, Scott & James for ICE Note 372 analysis and for 60N/mm² it is 72% greater. Therefore it is questionable whether tension zone stiffening can be relied on to the extent assumed in EC 2. It should also be noted that the quoted concrete tensile strengths are based on mean concrete tensile strength, without any safety factors, and the question of whether a section is cracked can also be affected by factors such as construction and loading history.

Taking these factors together, the EC 2 assumptions on concrete tension zone stiffening appear to be optimistic, particularly where high strength concrete is used.

UK National Annex to Eurocode 2

In the UK National Annex [13], Table NA.5 includes 'Note 5', which modifies EC 2 equations 7.16 and 7.17:

'The ratio of area of reinforcement provided to that required should be limited to 1.5 when the span/depth ratio is adjusted. This limit also applies to any adjustments to span/depth ratio obtained from Expressions (7.16a) or (7.16b) from which this table has been derived for concrete class C30/37'.

The meaning of the first sentence of Note 5 is clear enough: in Eq. 7.17, A_{sprov}/A_{sreq} should be limited to 1.5. However this is wrong, as the results would vary depending on the yield stress of the steel. The limit should be applied to the calculated ratio $310/\sigma_s$, not to A_{sprov}/A_{sreq}. On its own, this is a relatively minor problem but unfortunately, the second sentence of Note 5 does not make sense either. In EC 2 Equations 7.16a and 7.16b, the basic span/effective depth factor for a simply supported beam is K =1 and this is multiplied by a modification factor which varies with concrete strength and reinforcement ratio and is typically between 15 and 30. This cannot possibly be limited to 1.5, as required by Note 5.

Therefore it is impossible to design reinforced concrete beams and slabs to comply with EC 2 eq. 7.16 and the current UK National Annex.

Since this paper was submitted for publication a draft amendment to the UK National Application Document has been published for comment. This proposes a revision to Note 5 so that the limit of 1.5 is applied to $310/\sigma_s$, as proposed above, which would remove the steel service stress anomaly in the present NA. It also proposes to remove the unworkable limitation on the application of Eq. 7.16. However the proposed amendment would do nothing to limit the very high span/depth ratios permitted by Eq. 7.16 when concrete strength is high or M/bd^2 is low.

Design loadings

In current UK practice, deflection is normally checked at the full working or 'characteristic' load. However Eurocode 0 Clauses 1.5.3.17, 1.5.3.18, 4.1.3 and 6.5.3 define two other loading conditions: a 'frequent' loading and a 'quasi-permanent' loading. According to Eurocode 2 Cl. 7.4.1, deflection calculations should be based on the 'quasi-permanent' loading. (This is inconsistent with the UK National Annexes for Eurocodes 3 (steel) and 5 (timber), which state that deflection should be checked under the full characteristic service load.)

Eurocode 0 Table A1.1, states that the quasi-permanent load can be assumed to be the following proportions of characteristic imposed loads:

- roof, snow and wind loads: 0%,
- domestic and offices: 30%,
- retail, assembly and vehicles: 60%,
- storage: 80%.

Like many of the innovations in Eurocodes, this is a plausible-sounding idea which has not been properly thought through. If a concrete beam carries almost entirely permanent loads, it may not make much difference to the design if we ignore deflection caused by short term live loads. However is it appropriate to follow the advice of Eurocode 0 and completely ignore deflections caused by wind, snow and live loads when designing a lightweight roof structure? Is it sensible to completely ignore the deflection caused by lateral wind load when designing a structural frame?

Even if the principle of using a reduced imposed loading when checking deflection is accepted, caution would be in order: if reduced loading is applied with the same deflection limits as before, this would have the effect of increasing structural deflection compared with past practice.

Table 9 shows the service stress for typical structures under 'quasi-permanent' loading, compared with the nominal EC 2 service stress of of 310N/mm^2 under full characteristic load.

Table 9 Eurocode 2 'Quasi-permanent loading'

Usage/ Loading kN/m²	DL	LL	DL+LL	γDL	γLL	γDL+ γLL	DL+ ψ_2LL	f_s	f_s/310
Roof	4	0.75	4.75	5.4	1.13	6.53	4	267	0.86
Domestic	4	1.5	5.5	5.4	2.25	7.65	4.45	253	0.82
Domestic	7	1.5	8.5	9.45	2.25	11.7	7.45	277	0.89
Office	5	2.5	7.5	6.75	3.75	10.5	5.75	238	0.77
Office	8	2.5	10.5	10.8	3.75	14.55	8.75	262	0.84
Retail	5	4	9	6.75	6	12.75	7.4	252	0.81
Retail	8	4	12	10.8	6	16.8	10.4	269	0.87
Storage	5	7.5	12.5	6.75	11.25	18	11	266	0.86
Storage	8	7.5	15.5	10.8	11.25	22.05	14	276	.89

As can be seen from the table, changing from characteristic load to quasi-permanent load typically reduces the maximum steel service stress from 310N/mm² to 240-275N/mm². This would also reduce the 'required steel percentage' in EC 2 Cl. 7.16 correspondingly. If the 'quasi-permanent' steel service stress is (275/310) times the stress under 'characteristic' load, the EC 2 allowable span/effective depth ratios (based on 'Interpretation B' of equations 7.16) would then be as shown in Table 10.

Table 10 Allowable span/effective depths: 'quasi-permanent' loading

Total characteristic service loading	5kN/m²	10kN/m²	20kN/m²
BS 8110 (characteristic load)	28	24.6	21.5
EC 2 (f_{cu} = 30N/mm²) (characteristic load)	36.1	27.9	21.8
EC 2 (f_{cu} = 30N/mm²) (quasi-perm. load)	38.9	30.1	23.6

As can be seen from the table, checking deflection on the basis of 'quasi-permanent' loading rather than full characteristic loading would have the effect of increasing allowable span/depth ratios by 8%. If this is done, even at normal concrete strengths EC 2 would allow slabs supporting light and medium loadings to have span/depth ratios 20-40% greater than current UK practice.

Conclusions

The importance of span/depth rules for controlling deflection in reinforced concrete design is often underestimated. The most economical design for a slab will depend on loading, layout, relative costs of materials etc., but in most cases slabs should be made as thin as the deflection limits allow. However if design rules allow excessive deflection, the result can be sagging floors, cracked partition walls and an unhappy building owner.

The engineer needs to be able to determine the correct slab thicknesses and beam depths early in the design process, as these are difficult to change later. Therefore we need code of practice span/depth rules that are simple, able to be applied at the start of the design process and give reliable, sensible results.

The 2005 amendment to BS 8110 span/depth recommendations (Table 3.10) contains an error: the formula for f should have been revised following the change in steel material factor from 1.05 to 1.1 5. Rather than $0.667f_y$ = 333N/mm², it should be f_s =

$0.6f_y$ = 300N/mm^2. Correcting this error would allow thinner slabs and more economical concrete structures.

The recommendations in Eurocode 2 take into account the results of recent research, so they should be more accurate than BS 8110. However their presentation leaves much to be desired. What is needed is a table of recommended span/depth ratios based on slab type and loading (similar to IStructE 'Gold Book' Table 6c [10]) so that economical design schemes can be produced quickly and easily.

The EC 2 'simple' method for span/depth ratios in Cl. 7.4.2 Table 7.4N is easy to use but if guesswork is required when designing a structural scheme in most situations it produces over- conservative, uneconomical results. (Unfortunately, the IStructE/ICE Manual to EC 2 [14], to which most engineers will turn if they are asked to design a project to EC 2, only includes span/depth rules based on Table 7.4N.)

EC 2 Cl. 7.4.2 does also offer an alternative method, where allowable L/d limits are calculated using equations 7.16 and 7.17. Unfortunately, as currently draughted, this part of EC 2 suffers from serious practical and theoretical problems.

(a) Equations 7.16 and 7.17 can only be applied at the end of the design, after the reinforcement has been designed, so for scheme design the engineer is forced to rely on guesswork.

(b) According to equations 7.16(a) and (b), increasing concrete strength has a major effect on slab deflection. However this relies heavily on concrete tension zone stiffening, with tensile stresses which are substantially higher than recommended by recent research, which shows that concrete tensile resistance reduces rapidly under sustained loading. It therefore appears that the EC 2 span/depth ratios are excessive for lightly-loaded beams and slabs, or when high strength concrete is used.

(c) It is not clear how Eq. 7.16(a) and (b) are intended to be applied when the reinforcement service stress varies from 310N/mm^2 as it is not clear how the reinforcement ratio ρ in eq. 7.16 is to be calculated. Is it *the actual amount of reinforcement present*, or is it *the reinforcement which would have been required for a design stress of 310N/mm²*? As currently drafted, it is not clear which of these interpretations is correct.

(d) Not only is Eq. 7.16 ambiguous for steel stresses other than 310N/mm^2 but analysis reveals that there are problems with both of the possible interpretations. If ρ is the actual steel ratio (interpretation 'A', $\rho = A_{sprov}/bd$), then the allowable L/d is almost constant for f_s down to 200N/mm², so if a slab or beam fails a span/depth ratio check, increasing the reinforcement will make no difference. This cannot be correct. On the other hand, if ρ is supposed to be the theoretical steel ratio which required for a steel service stress of 310N/mm² regardless of actual stress (interpretation 'B', $\rho = (\sigma_s/310) \times (A_{sprov}/bd)$), then steel stress has more effect on the allowable L/d ratio than in BS 8110. EC 2 Eq. 7.17 assumes that doubling the steel reinforcement will reduce the deflection by half.

However this is incorrect, because of concrete tension zone stiffening and neutral axis shift, the stiffness of a reinforced concrete beam does not vary in proportion to the area of tensile reinforcement. As a result, Eq. 7.17 exaggerates the effect that varying f_s has on the allowable span/depth ratio.

(e) The UK National Annex modifies the EC 2 recommendations to try to minimise problems (b) and (d). However unfortunately, as currently written, the section relating to EC 2 Eq. 7.17 contains a logical error and the section relating to Eq. 7.16 imposes a limit with which it is impossible to comply. As a result the UK NA is unusable in its present form. A draft revision to the UK NA which has been published would solve the problem with Eq 7.17. However it would still leave the problem of excessive span/depth ratios in EC 2 when high strength concrete is used.

(f) In addition to its new span/depth rules, EC 2 also proposes that deflection should be calculated using a reduced 'quasi-permanent' loading. If this is done, EC 2 would consistently permit greater span/depth ratios than current UK practice. Without good evidence that these shallower beams and slabs will perform satisfactorily, there is a risk that there could be deflection problems. Problems may also arise from the EC 2 proposal to allow deflection caused by roof live loads and the lateral wind deflection of frames to be completely ignored in serviceability design. To avoid these problems, it would be prudent for the UK National Annex for EC 2 to adopt the same approach as those for EC 3 (steel) and EC 5 (timber) and require deflection to be checked under full characteristic loads.

Clearly a substantial amount of research would be required to explore and resolve all the anomalies and problems which have been identified and this is outside the scope of this paper. However, until this is done it would be prudent to take a conservative approach in the UK National Annex and to press strongly for early amendments to EC 2 to remove the ambiguity in its recommendations and rectify the most obvious errors. In the circumstances it would be helpful if BSI could reconsider its decision to declare BS 8110 'obsolescent' and issue an amendment to correct the present error in the steel service stress formula in Table 3.10.

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References

1. BS EN 1992-1-1:2004 *Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings,* British Standards Institution, London, 2004, revised 2008
2. BS 8110:1985, *Structural Use of Concrete,* British Standards Institution, London, 1985
3. CP 114:1969, *The Structural Use of Concrete in Buildings,* British Standards Institution, London, 1969
4. CP 110: Part 1: 1972, *The structural use of concrete,* British Standards Institution, London, 1972
5. Beeby, A. W.: *Modified proposals for controlling deflections by means of ratios of span to effective depth,* Technical Report 456, Cement & Concrete Association, Wexham Springs, 1971
6. Beal, A. N.: 'Span/depth ratios for concrete beams and slabs', The *Structural Engineer,* 61A/4, April 1983, pp 121-123, Institution of Structural Engineers, London
7. Rowe *et al, Handbook to British Standard BS 8110:1985 Structural Use of Concrete,* Palladian Publications, London, 1987
8. Beal, A. N., Beale, F: 'Control of deflection in reinforced concrete', 'Verulam', *The Structural Engineer,* **62A/** 3, March 1984, Institution of Structural Engineers, London
9. Institution of Structural Engineers/ Institution of Civil Engineers *Manual for the design of reinforced concrete building structures,* 2nd Edition, Institution of Structural Engineers, London 2002
10. Institution of Structural Engineers, *Recommendations for the permissible stress design of reinforced concrete building structures,* Institution of Structural Engineers, London, 1991, revised 2002
11. Goodchild, C.: *Economic Concrete Frame Elements,* Publication 97.358, British Cement Association, Crowthorne, 1997, p. 15

12. Beeby A. W., Scott R. H., Jones A. E. K.: 'Revised code provisions for long term deflection calculations', *ICE Proc., Structures & Buildings,* **158/** 1, February 2005, pp. 71-75, Institution of Civil Engineers, London, 2005

13. NA to BS EN 1992-1-1:2004, *UK National Annex to Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings,* British Standards Institution, London, 2005

14. *Manual for the design of concrete building structures to Eurocode 2*, Institution of Structural Engineers, London, 2006 (revised 2008)