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Span/depth ratios for concrete beams and slabs

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Synopsis

While the treatment of deflection in CP110 has been generally welcomed as an improvement on the rather rough-and-ready span/depth rules in CP114, it is very cumbersome to apply in practice. The designer cannot check the span/depth ratio until section design is almost complete and, if a problem arises, he/she has little guide as to what scope there is for solving it by redesigning with a reduced steel stress.

By retabulating CP110's modification factors in terms of M/bd^2, *rather than* A_s/bd, *it is possible to simplify the presentation considerably and to separate the effect of steel design stress front of varying* M/bd^2. *This allows span/depth ratio to be checked earlier in a calculation and also clarifies the effect of designing to different steel stresses. Tables are presented for determining span/depth ratios for designs to CP114 and CP110 and a table of approximate span/depth ratios for the preliminary design of slabs is also presented.*

Introduction

Deflection control in concrete beams and slabs is an approximate business that has traditionally been covered by the application of span/depth ratios. In CPI 14[1], this is a simple matter - the appropriate span/overall depth ratio is selected from Table 13, depending on end or edge conditions, steel and concrete stress and on whether a beam or slab is being designed. This approach gives satisfactory results in most cases; however, in certain circumstances, there have been problems and, as a result, CP110[2] introduced a new, more complex approach. In CP110, basic span/effective depth ratios are still quoted (Tables 8 and 9) but these are then modified by an array of factors that relate to steel stress and quantity of steel. There is a further table of factors to cover the effects of any compression steel present (Table 11).

The CP110 approach has generally been welcomed as more correct but it is a very cumbersome process to use in design. Ideally, the required span/effective depth should be available at the start of the calculation so that the correct section size can be selected at the outset and the design is both speedy and economical. At the expense of being rather rough-and- ready, CP114 does achieve this. However, in CP110, the permitted span/effective depth is known only when the steel area provided and the steel stress are known - thus it can be checked only when section design is virtually complete. In practice, this means that the designer labours over two or three redesigns of the section for optimum results or else a needlessly over-conservative approach is followed - it is quite common now to see slabs designed quite unnecessarily to a span/effective depth of 20 simply in order to avoid trouble arising later in the design. Because reduced steel stress increases the allowable ratio but the accompanying increase in steel area acts to reduce the allowable ratio, it is also far from clear to the designer what scope (if any) there is for solving a deflection problem by designing to a reduced steel stress.

An approach that allowed section depth to be decided early in the calculation and that made clear the relationship between steel design stress and allowable span/effective depth would be a distinct improvement.

How does span/depth ratio control deflection?

For a symmetrical, elastic beam supporting a distributed load, the deflection can be calculated purely from the extreme fibre bending stress, the section depth, and the span. If the permissible bending stress is known and the deflection limit is some proportion of the span (such as L/360), then a constant span/depth ratio can be established that will ensure compliance with this limit. The span/depth limit varies directly with the bending stress.

Steel beams can be designed in this way, and the appropriate span/depth ratios are tabulated in the BCSA/Constrado Handbook [3] (Table, p. 16). However, the situation in reinforced concrete is more complex:

* it does not behave in a strictly elastic fashion;
* the neutral axis depth is not constant but varies with the quantity of reinforcement;
* although concrete in the tension zone contributes little to ultimate strength, it can substantially reduce deflections.

In these circumstances, the factors given in CP114, Table 13 can be regarded as only very approximate; the varying factors in CP110 Table 10, are intended to cover the possible variations in a much more thorough fashion. However, as stated earlier, this is achieved only at the expense of great inconvenience in design.

CP110 span/effective depth ratios simplified

The factors given in CP110 Table 10, depend on steel stress and steel area. For a rectangular section it is possible to recalculate these and present them in terms of M/bd^2, rather than $100A_s/bd$ for a given steel stress. (The factors are calculated from the formula $1/(0.225 + 0.00322f_s - 0.625 \log (bd/100A_s))$ with a limit of 20 applied, where f_s is the steel tensile stress.) The results are given in Table 1 for 140N/mm² (CP114 mild steel), 145N/mm² (CP110 mild steel), 230N/mm² (CP114 high yield steel), and 267N/mm² (CP110 high yield steel). M is the working (unfactored) moment but the ultimate (factored) value can be taken as 1.5 x this.

Table 1

Steel stress (N/mm²) M/bd^2 (N/mm²)	0.5	1	2.0	3.0	4.0
140	2.00	1.67	1.25	1.07	0.96
145	2.00	1.65	1.24	1.07	0.96
230	1.77	1.33	1.04	0.92	0.83
267	1.55	1.20	0.96	0.86	0.78

If we take out factors of 1.25, 1.24, 1.04 and 0.96, from the values for 140N/mm², 145N/mm², 230N/mm², and 267N/mm², respectively, the results are as in Table 2.

Table 2

Steel stress (N/mm²) M/bd^2 (N/mm²)	0.5	1	2.0	3.0	4.0
140	1.60	1.34	1.00	0.86	0.77
145	1.62	1.33	1.00	0.86	0.77
230	1.70	1.28	1.00	0.88	0.80
267	1.62	1.25	1.00	0.90	0.81

It can be seen that one set of factors could be used for all steel stresses with little error, with basic span/depth ratios being quoted for the main design steel stresses. This would allow the section to be selected before designing the reinforcement and would show clearly the effect of varying the design steel stress.

Compression reinforcement is rarely used to control deflection; it is almost always used as a means of boosting the resistance moment of the section when reinforcement is heavy. CP110 Tables 10 (tension steel) and 11 (compression steel) reveal that when more than 0.75% high yield tension steel is present, any reduction in the factor caused by an increase in the tension steel would be approximately cancelled if a corresponding quantity of compression steel was introduced. Thus for 0.75% tension steel at 238N/mm² (f_y = 410N/mm²), the factor is 1·09; for 2% tension steel plus 1.25% compression steel, the factor would be 0.84 x 1.29 = 1.08. Similarly, for 1.5% steel at 238N/mm², the factor is 0.9; for 2% tensile steel and 0.5% compression steel, the factor would be 0.84 x 1.14 = 0.96. When we remember that the use of compression steel as a means of controlling deflection is very rare (and expensive!), it can be seen that it would be quite adequate in these cases to calculate the effective M/bd^2 for deflection as $(M-M_c)/bd^2$ where M_c is the moment resistance of the compression steel. This is quick and accurate enough for all normal purposes, provided the resulting effective M/bd^2 is not taken to be less than 1.5. CP110 Table 11 may still, of course, be used instead, if this is more convenient.

The second consideration with compression steel is that any heavily reinforced beam is certain to have links and thus some threader bars in the compression zone. Thus, while a span/depth ratio for a singly-reinforced beam with M/bd^2 = 4 has been presented, it is really of only academic interest and might be omitted from practical tables. However, it could still be necessary if the effect of compression steel is being calculated using CP110 Table 11 and so it is included in brackets in the tables that follow.

The approach outlined gives results that agree closely with CP110's requirements and thus Tables 3 and 4 can be used directly for design in place of CP110, Tables 8, 9, 10, and 11, with definite advantages in convenience and speed for the designer.

Table 3 - Basic span/depth ratios for CP110 design

f_y	250N/mm²	425N/mm²	460N/mm²	485N/mm²
Cantilever	8.7	7.0	6.7	6.5
Simply supported	24.8	20.1	19.2	18.6
Continuous	32.2	26.1	25.0	24.2

NOTE: These should be reduced for spans over 10m by a factor of 10m/span.

Table 4 - Modification factors for CP110 design (ultimate moments)

M_u/bd^2	≤0.75	1.5	3	4.5	(6.0)
	1.6	1.3	1.0	0.9	0.8

NOTE: Where compression steel is present, its resistance moment may be deducted in calculating M_u/bd^2 for deflection, provided the resulting M_u/bd^2 is not less than 2.5. Alternatively, CP110 Table 11 may be used.

CP114's requirements are quick and easy to apply as they stand, but they are regarded as suspect in some cases and may be over-restrictive in others. Tables 5 and 6 provide a convenient means of checking a CP114 design to the CP110 criteria.

Table 5 - Basic span/effective depth ratios for CP114 design

Steel stress	140N/mm²	210N/mm²	230N/mm²
Cantilever	8.8	7.6	7.3
Simply supported	25.0	21.7	20.8
Continuous	32.5	28.2	27.0

NOTE: For spans over 10m, these should be reduced by a factor of 10m/span.

Table 6 - Modification factors for CP114 design (working moments M)

M/bd^2	≤0.5	1.0	2.0	3.0	(4.0)
Factor	1.6	1.3	1.0	0.9	(0.8)

NOTE: Where compression steel is present, its resistance moment may be deducted in calculating M/bd^2 for deflection, provided the resulting M/bd^2 is not less than 1.5. Alternatively, CP110 Table 11 may be used.

T-beams and ribbed slabs

As the foregoing consists only of a revised presentation of the data in CP110, it may be applied directly for design to that Code. However, there is one other aspect of deflection design to consider - and here it may be possible to improve on the CP110 approach. This concerns T-beams and ribbed slabs.

CP110 clause 3.3.8.2 requires the permitted span/effective depth ratio to be reduced to 0.8 of the normal value for beams with rib widths b_r less than 0.3 of the flange width b. This constant reduction is unlikely to be completely correct for both lightly-reinforced beams (where concrete stiffening is important) and heavily-reinforced beams (where it is not).

Three things affect deflection:

- neutral axis shift with varied % steel;
- shrinkage curvature;
- stiffening due to concrete in the tension zone.

(1) and (2) are more or less independent of b_r/b (providing the neutral axis is in the flange). (3) can be considered to be directly proportional to b_r/b. Thus if values are tabulated for $b_r/b = 0$ and $b_r/b = 1$, intermediate values may be obtained by linear interpolation.

In the paper where the CP110 span/depth tables were derived, Beeby [4] calculated the effect of ignoring concrete tension zone stiffening. The effect is to reduce the multipliers by a factor of between 0·75 (0.25% steel) and 0·98 (3% steel). With the simplified presentation given here, these can now be incorporated in practical tables, along with those calculated earlier. They are presented in Table 7.

Table 7 - Modification factors for $b_r/b = 1$ and $b_r/b = 0$

CP114	M/bd^2	0.5	1.0	2.0	3.0	(4.0)
CP110	M_u/bd^2	0.75	1.5	3	4.5	(6)
Factors	$b_r/b=1$	1.6	1.3	1.0	0.9	(0.8)
	$b_r/b=0$	1.2	1.1	1.0	0.9	(0.8)

Intermediate rib widths can be interpolated.

As can be seen, the CP110 reduction factor of 0.8 for $b_r/b = 0.3$ is reasonable for the lower range of values, but over-conservative for higher ones ($M/bd^2 > 1{\cdot}0$). With the simpler presentation given here for modification factors, it is quite easy for this more accurate treatment to be presented and used. It would seem to be an improvement on the method given in CP110.

Span/effective depth ratios for preliminary design

Deflection governs the design of beams only on some occasions and for these the basic ratios quoted should be satisfactory for the preliminary design. However, the design of slabs is almost always governed by deflection and it is essential to be able to estimate slab thicknesses early in the preparation of a scheme. While the approach set out in this paper is more convenient for design than is that in CP110, it still lacks the simple directness of the slab span/thickness ratios in CP114 for preparing a structural scheme. The slab thickness depends on the steel design stress and also on the applied loading.

Table 8 gives approximate span/effective depth ratios for different forms of construction and loading for the purposes of preliminary design. Slab thicknesses determined from these should require little adjustment in the final design. They have been calculated for spans up to 10 m, both for a 'light' imposed loading of 2.5kN/m² (equivalent to domestic loading plus light partitions on a directly finished slab) and for a 'heavy' imposed loading of l0kN/m² (equivalent to 7.5kN/m² stockroom loading plus 5kN/m² screed plus light partitions). The values for continuous slabs are based on moments from CP114 Table 15, and those for two-way slabs and flat slabs are based on moments from CP110 Tables 12, 13 and 18. It is assumed that (a) steel strength is 460N/mm² or 425N/mm² according to bar size and (b) cover is 15mm or bar size, whichever is greater. Values were calculated for 100mm, 200mm and 300mm slabs and the results shown are rounded averages of the permissible span/effective depth ratios determined in accordance with Tables 3, 4, 5 and 6 of this paper. Allowing for the difference between overall and effective depth, it can be seen that the ratios are quite close to CP114 values, being slightly lower for simply supported and higher for continuous slabs.

Table 8
Approximate span/effective depth ratios for preliminary design of slabs

Steel service stress (N/mm²)	Super-imposed loading (incl. finishes) (kN/m²)	Simply supported	Continuous	Cantilever	Two-way ss	Two-way cont.	Flat Slab no drops	Flat Slab drops
230/210(CP114)	2.5	29	39	11	33	43	39	42
	10.0	25	34	10	29	41	35	37
267/247(CP110)	2.5	28	37	11	31	41	37	40
	10.0	24	32	10	28	39	33	35

NOTES:

- Two-way slabs have been calculated for a square panel. For a 2 x 1 panel, the value for a one-way panel should be used and values interpolated for intermediate proportions.

- Flat slab design should be based on the longer span dimension. For exterior panels spanning to walls, use 85%-90% of the quoted ratio.
- For ribbed slabs, use 85%-90% of the quoted ratios.
- For design with mild steel stresses, ratios can be increased by 15%.

Acknowledgements

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References

1. CP114 *The structural use of reinforced concrete in buildings,* London, British Standards Institution, 1969.
2. CP110 *The structural use of concrete: Part 1,* London, British Standards Institution, 1972.
3. *Structural steelwork handbook - properties and safe load tables*, BCSA & Constrado, London, 1978.
4. A. W. Beeby: 'Modified proposals for controlling deflections by means of ratios of span to effective depth', Technical Report, Cement & Concrete Association, April 1971.

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Control of deflection in reinforced concrete

A short paper 'Span/depth ratios for concrete beams and slabs', by Mr Alasdair Beal, published in The Structural Engineer *for April 1983, dealt with the treatment of deflection in CPs 110 and 114. Mr Francis Beale has written to us expressing great interest in the paper and suggesting that some modifications were required to Table 8. In his letter, which is quoted below, Mr Beale provided a revised table showing generally lower values for recommended span/depth ratios, together with some further comments:*

I have taken the opportunity to modify some of the figures to reflect the endspan or corner panel condition in all cases, so that a ready design tool is available and in the case of flat slabs, I have assumed support by columns.

My attention has been drawn to the need for a meaningful comparison between CP114 and CP110 by the seeming impossibility of designing flat slabs to CP110 and getting the sort of results one is used to. A flat slab designed to CP114 requires a 250 slab but that for CP110 requirements would be 300 thick using 460 steel.

I have used a loading of 10 kN/m² for comparison purposes because it is reasonably common, is permitted by CP114 (i.e. any loading) and eliminates the distortion in lower loadings caused by the cutoff point of CP110 tables.

It will be noted that, in all cases, CP110 is more onerous and the effect can be very large (25% increase in thickness of slab) for two-way, simply supported slabs.

While writing, I think it is time someone mentioned the practical effect of Table 19, 'Nominal cover to reinforcement' of CP110. In order to get a sensible cover in slabs (15 mm) one is required to use grade 30 concrete, effectively as a minimum grade of concrete whether required from other considerations or not. A very large number of jobs can be designed using 21N/mm² concrete. When other considerations are taken into account, this means that a typical flat slab contract will use about 40N/mm² more cement when designed to CP110.

Mr Beale concludes his letter by asking whether it is time for CP114 to be discarded. We referred the points raised to Mr Beal for comment. He replied as follows:

(1) The lower span/depth ratios Mr Beale calculates arise mainly from his use of 'endspan' and 'corner bay' conditions for continuous slabs. The values for lower loadings are also affected by the fact that CP110 Table 10 gives multiplier values only down to steel percentages of 0.25%, although the multiplier limit of 2.0 is reached only at much lower percentages than this with high yield steel. Using the true values for less than 0.25% steel gives rather better results in many cases.

(2) It is a good question whether ratios for continuous slabs should be based on internal or edge bays. Test calculations on these show that the appropriate ratios for endspans and corner bays of continuous slabs are between 87% and 93% of those for internal spans. The best solution may be to tabulate values for internal spans, with a note to the effect that the ratios for endspans and corner bays of continuous slabs may be taken as 90% of these, with negligible error.

TABLE V1

Proposed span/effective depth ratios for slabs up to 10 m span

Steel service stress	Total service load (kN/m²)	Simply supported	Continuous	Cantilever	Two-way ss	Two-way cont.	Flat slab	
							No drops	Drops
230N/mm² (CP114)	5	32	44	13	37	51	43	46
	10	27	37	11	32	46	37	41
	20	24	32	10	27	39	32	34
267N/mm² (CP110, *fy* = 460N/mm²)	5	30	40	12	35	50	41	44
	10	26	35	10	30	43	35	38
	20	23	30	9	26	37	30	32

(3) In comparing CP110 with CP114, Mr Beale's imposed load of 10 kN/m² is high and an thickness/effective depth ratio

of 1.15 is more appropriate.

(4) It would probably be better if, as Mr Beale suggests, exact rather than approximate span/depth ratios could be quoted. This can be done if the table is presented in terms of total rather than imposed slab service load; Beeby [1] has presented proposals of this sort and they were included in the draft revised CP110 [2]. However, these are still rather awkward to use. The best solution may be to tabulate span/depth ratios for total (dead + live) loads of (say) 5, 10, 20 kN/m² at the preferred steel service stress. The values calculated are presented in Table Vl.

Notes:

(i) Two-way slabs have been calculated for a square panel. For a 2 x 1 panel, the value for a one-way panel should he used and values interpolated for intermediate proportions.

(ii) Ratios for all continuous slabs are for internal bays. For endspans and corner bays, the ratios should be reduced to 90% of stated values.

(iii) For design with mild steel stresses, ratios may be increased by 15 %. For steel with a yield stress of 425N/mm², with a service stress of 210N/mm² (CP114), 247N/mm² (CP110), appropriate ratios may be increased by 3%.

(iv) For ribbed slab, the ratio should be reduced by 85-90%, depending on rib width.

(v) Flat slab design should be based on the longer panel dimension.

(vi) For loadings and arrangements not covered, design should be based on Tables 3 or 5 and 7 in the original paper.

(5) If we take a total (dead + imposed) slab load of 10kN/m² as typical and a thickness/effective depth ratio of 1.15 and consider corner bays and endspans, as Mr Beale suggests, then a comparison with CP114, with mild steel stress (140N/mm²), gives the results shown in Table V2 for span/thickness ratios.

As can be seen, differences are slight. If, as is usual, CP114 values for high yield steel are taken as 85 % of mild steel values, the differences here will be small also. (Flat slabs to CP114 are an anomaly, where no allowance seems to have been made for increased steel stresses.) However, as Mr Beale points out, CP110 values become more conservative under heavy loads.

Table V2

	Simply supported	Contin-uous	Cantilever	Two-way ss	Two-way cont.	Flat slab	
						No drops	Drops
Table V1	27	33	11	32	41	33	37
CP114	30	35	10	35	40	32	36

(6) The effects of CP110 on slab thickness, cover and concrete mixes mentioned by Mr Beale raise several fresh issues, some outside the scope of the paper. Some of the changes in CP110 are understandable but others are not - thus its increases in slab thickness are mainly required to balance the effect of increased steel service

stresses on deflection. These increased steel service stresses also increase the cover required for adequate fire resistance - thus CP110's savings in steel are offset, to some extent, by increases in concrete. However, CP110's insistence on increased nominal cover with 21N/mm² concrete used indoors is hard to understand, in view of the lack of observed service problems (probably the majority of slabs in use have 1:2:4 (21N/mm²) concrete, with ½in (13 mm) cover. The draft revision of CP110 went further - it has been pointed out that this would effectively ban 1:2:4 concrete in structural work!

Should CP114 be discarded? That is a much wider debate, involving many questions, some of which have been discussed elsewhere [3]. If the proposals presented here are accepted, they may be used in both CP110 and CP114 .

References

1. Beeby, A. W.: 'Span/effective depth ratios: a transformation of the CP110 method', *Concrete,* 13, No. 2, Feb. 1979.
2. CSB/39 *The structural use of concrete,* London, British Standards Institution, February 1982.
3. Beal, A. N.: 'What's wrong with load factor design?', *Proc. ICE,* Part 1, November 1979.