What's wrong with load factor design?

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Load factor analysis is now being adopted as the standard approach for all structural civil engineering codes of practice. When the new codes have been criticized, this has generally been on the grounds of their practicality rather than their theoretical basis. After summary of some of the practical problems, the load factor approach is shown to have theoretical weaknesses in several areas, and the attempt to define structural design strictly in terms of probability theory is shown to have serious limitations. In particular, the analysis of dead load variations proposed in CP 110 is shown to give highly inconsistent and misleading results. In view of the problems of the load factor approach, its complexity seems to be unjustified. A return to stress factor analysis, based on working loads, is proposed and the considerations involved are discussed.

Introduction

The 1970s have seen a revolution in structural engineering codes of practice: a wholesale revision of existing codes has taken place, with the traditional approach based on working stresses giving way to what is known as 'limit state theory'. The principal difference is the replacement of the 'stress factor' approach to safety margins by a 'load factor' approach. The stress factor approach is still the most widely used and there is no doubt that a full-scale transfer to load factor analysis would require great changes in thinking and practice.

2. In structural analysis, forces generated by estimated loads are compared with the forces required for structural collapse. Generally, stress theory is used to translate the applied loadings into stresses which can then be compared with the stresses associated with failure. The probability of failure can be kept to a suitably low level either by the application of a reduction to the stresses permitted in design ('stress factor'), or by application of a factor to increase the apparent loadings before comparison of estimated stresses with failure stresses ('load factor').

3. In this Paper, the term load factor is restricted to the meaning described here, to make a clear distinction between the two approaches. On this basis, the method referred to in CP 114 as load factor would fall into the category of stress factor methods as it is normally applied with working loads. Although
the new codes also incorporate factors on material strengths, this Paper is concerned with points of difference in approach, and so it concentrates on the load factor aspect.

4. The stress factor approach is elegant and simple—the stresses caused by applied loads can be compared directly with tabulated permissible stresses, producing structures with a consistent safety margin in a straightforward fashion. Load factor design is more complicated, as various factors must be applied to the loads (and sometimes also to the allowable stresses) before the actual and permitted stresses can be compared.

5. The advantage proposed for the load factor method is a greater theoretical rigour and exactness in dealing with possible variations in loadings and strengths, allowing full application of probability theory to structural safety. Codes currently appearing have generally attempted to tailor factors so that the resulting structures differ little from those designed to previous codes, without substantial difference in overall factors of safety. While this would appear to render the extra complexity of analysis rather unnecessary, the idea is that, by erecting a theoretically rigorous framework for analysis and gaining experience in its use, it will be possible to incorporate advances in knowledge of materials and loadings confidently to produce economical, safe structures, fully justifying the increased design effort required. Clearly the concepts involved are more applicable to some materials (e.g., steel, reinforced concrete) than to others (e.g., timber, masonry) but all structural engineering codes are being revised in the interests of consistency.

6. Such a fundamental change has aroused considerable controversy and criticism. However, this has tended to be on the basis that it is all very well in theory, but impractical in use, and that as millions of satisfactory structures have been built to designs based on the stress factor approach, is it really necessary to change?

7. The Author is one of the generation brought up on load factor design and he left university convinced of its merits. The criticism of it which follows results from his experience in applying both old and new codes in the design office.

8. This Paper sets out to demonstrate that not only is the load factor method cumbersome to use, but it actually has substantial theoretical flaws.

9. So far, only one limit state code, CP 110, has been used to any great extent; other limit state codes which have appeared are BS 5628 Part 1 (unreinforced masonry), the Merrison Committee's interim design rules for steel box girder bridges (IDR) and some sections of BS 5400 (bridges). Although limit state theory was intended as a unified consistent approach to structural design, each of these codes offers an approach which is different in some way from those of the others.

(a) BS 5628 offers slightly different load factors from CP 110.

(b) While CP 110 interposes only one factor, $\gamma_L$, between loads and permissible stresses (the load factor), BS 5628 interposes two (a load factor and a materials factor), and BS 5400 has no less than three (an analysis factor is added). Previous codes compared loads and permissible stresses directly.

(c) Dead load variations are dealt with in different ways. In CP 110 maximum and minimum factored values of dead load are applied.
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span-by-span so that they combine for worst effect, whereas in the IDR they are applied to appropriate segments of the influence line for worst effect. In BS 5400 dead load is considered to be increased throughout the structure or decreased throughout, without local variations. BS 5628 does not specify its approach clearly, but consistency with CP 110 is probably intended.

Because of these differences (particularly (c)), some of the points made in this Paper will not apply equally to all of the new codes.

General and practical objections to codes of practice based on the load factor method

10. The following is a summary of arguments raised in discussions of the new codes.

11. The first point is that design costs will be increased by the extra analytic work involved.

12. Secondly, if the results obtained from the new codes are similar to those produced by existing codes, the extra complexity demanded is unjustified, with little or no material savings to balance the increased design costs. Given the nature of the materials and loadings involved, it is unlikely that their characteristics will ever be known sufficiently accurately to bring any real benefit.

13. Thirdly, by demanding consideration of more than one value for each load, the new codes greatly increase the chance of calculation errors, with the possibility of unfactored loads being compared with permissible ultimate stresses. In CP 110, this would lead to approximately two thirds of the necessary reinforcement being provided; in the new masonry code, omission of material and load factors in a calculation could lead to errors of a factor of up to approximately 5. (If $\gamma_m = 3.5$, the factor is $3.5 \times (1.4DL + 1.6LL)$.)

14. Fourthly, although the new codes expound the value of deriving loads by statistical methods, the loads actually specified may not be arrived at in this manner. For example, clause 3.4 of BS 5628 reads as follows:

'Characteristic Load Ideally, where the load acts unfavourably, the load which has a probability of not more than 5% of being exceeded or, when the load acts favourably, the load which has a probability of at least 95% of being exceeded. In practice, the load obtained from the appropriate British Standard.'

The use of loads which are not statistically derived within a framework where everything else is might be decidedly risky; this problem does not arise in practice because, although the loads are not statistically derived, neither are the load factors applied. These have been set to give results broadly similar to those from existing codes. As this is so, an emphasis on statistically derived loads is confusing and misleading.

15. Fifthly, if analysis is made excessively complex, it will divert the engineer's attention from the all-important questions of structural form and detail.

16. The sixth argument is that most structural failures in practice are not collapses but are in fact serviceability failures (excessive deflexion, cracking of concrete or brick, or corrosion or fatigue of steel). When structures do collapse, this is almost always caused by serious errors in design or construction, or else
by extraordinary loads which bear little relation to normal imposed and permanent loads (e.g., collisions, extreme typhoons, earthquakes). The case of structural collapse brought about by excessive statistical variation of normal dead and imposed loads is a rare one and, by concentrating on it, analysis based on the ultimate limit state is giving it excessive emphasis.

Problems with load factors

17. The load factor method is proposed as a suitable vehicle for defining structural safety in probability terms. In design, the most important thing is the structure's factor of safety; this is the reserve of strength it has over the effects of the estimated applied loads. The end effect of all the arithmetic manipulations of the load factor method is to vary this factor somewhat, depending on the mix of applied loads: for practical reinforced concrete structures designed for normal dead and live loads to CP 110, it varies between 1.65 and 1.8 (approx.); for those designed to BS 5400, it may vary between 1.6 and 2.0 (approx.). Similar structures designed to CP 114 have a constant factor of 1.8. It is thought that variations of this kind are rational and will bring increased economy without loss of safety.

18. However, there are problems.

Engineer error

19. Applying probability theory to dead load analysis and structural strength implies that the efforts of concretors, steel fixers, carpenters, bricklayers etc. are satisfactorily defined in this way; if this is so, then the efforts of engineers should also be subject to this type of analysis. As the load factor method requires two sets of loadings for dead load rather than the single set considered by stress factor analysis, and it requires engineers to insert up to three factors before comparing applied loads with permitted stresses, it clearly increases the probability of engineer error, and rational values for safety factors must be increased to account for this.

Non-random variations

20. Probability theory assumes that variations in the events analysed are random and the events analysed are generally assumed to be independent of one another; if they are not (if cause and effect rather than random chance is at work), then it is not possible to define structural variation completely in terms of classical probability theory.

Partial safety factors

21. As the new codes propose a new approach to safety factors, it is worth looking at the differences involved and considering any shortcomings there may be. In the load factor codes, the safety factor for the structure is split into several parts, each with a separate function. The idea is that each can then be investigated and defined separately so that as a result, when they are recombined in analysis, a more rational value of overall safety factor is incorporated into the structure. The areas considered are

(a) unusual increases in a loading beyond the characteristic value;
(b) inaccurate assessment of the effects of loadings;
(c) variations in dimensional accuracy and material strength.
A separate factor may be applied for each of these. The strength of a material is taken as its 'characteristic' value reduced by some constant partial safety factor. Differences in the effects or probabilities of load combinations are taken up by variations in the values of load factors applied. For a given type of load within a combination, the maximum ultimate value is taken to be some constant multiple of the initial design value.

22. If the safety margin of the structure is to be split up in this fashion and an attempt then made to define each of these factors closely, one should first ask: 'Has any function of the safety factor been omitted?'

23. The safety factor for a structure does have other functions. In addition to the points mentioned, it serves to provide a reserve of strength against deterioration or corrosion, damage, alterations to the structure, and the effects of loads which are quite unexpected in size or nature. Also, it serves to limit service stresses to acceptable values. (This is still required, as limit state codes normally require analysis for only one limit state.)

24. It should next be asked: 'Does this approach satisfactorily analyse and combine the effects considered?'

25. Again it seems inadequate in some respects. Firstly, some materials have different capabilities for different types of load. Timber strength varies with loading duration; high permanent loads produce creep in timber and reinforced concrete (in RC this can produce severe cracking in the long term); in steel, high load fluctuations give rise to fatigue, so steady loads are more easily sustained than varying ones.

26. Secondly, changes in some loads during the life of the structure (particularly secondary dead load) may be quite unrelated to the original design value. The safety margin to be provided in such a case cannot really be considered as some specified multiple of the load estimated initially. Safety is more likely to be ensured in these cases by taking care to select suitable design loads, and by always ensuring that the structure has a reasonable overall reserve of strength.

27. Thirdly, even if each area considered has its probability of variation satisfactorily defined in isolation, the final safety of the structure results from the combination of all the applied safety factors. The combination of probability functions is a tricky process which can sometimes give results quite different from those expected (cf. §§ 39-46).

High dead load

28. Despite the points raised so far, the load factor approach may give some advantage in one area—it allows a lower safety factor to be allocated to structures which carry mainly dead load. If dead load is closely controlled, one may have more certainty about its value than about values of other loads, so this would seem reasonable. In the stress factor approach a constant factor is applied to both live and dead loads; if the proportion of dead load is high, this produces a very high real factor of safety on live load which may not be necessary. This may seem illogical, yet for many practical situations it produces sensible results.

(a) The types of structure which have a high proportion of dead load are often those where the consequences of failure would be unusually severe.
The elements within a structure which carry the highest dead load proportion are generally the most important. If these are the least likely of a structure's elements to fail, then this is not a bad idea.

Certain effects which lead to serviceability failure (e.g., creep and cracking in concrete) are principally generated by permanent loads.

Obviously these arguments do not apply in all cases and it could be beneficial to reduce the safety factor applied to known permanent loads. There is nothing unique in the ability of the load factor approach to achieve this. A stress factor code need only have inserted a clause of the following form: 'Where dead loads comprise more than 50% of the force in a member, quoted permissible stresses may be exceeded by up to 10% at the engineer's discretion.'

Dead load analysis

In the areas considered so far, the changes the load factor approach proposes are confined to arithmetic manipulation of safety factors. In fact there is only one area where any change is proposed in the nature of the loadings applied—this is dead load analysis. Instead of applying a constant value of dead load throughout, CP 110, BS 5628 and IDR apply maximum and minimum factors to it, as appropriate, for worst effect. Only continuous structures are affected; the intention is to allow for local dead load variations.

As this is a new approach, it is worth investigating closely.

What event is considered?

Central to probability theory is the idea of an event: if the event is not defined, then its probability cannot be defined either. If structural dead load variation is to be considered in probability terms, some element of the structure must first be selected to serve as the basic unit considered when dead load variation and its probability are being defined. What is this unit to be? In a concrete structure, is it the section cast by one pour of concrete, one unit of formwork, one batch of concrete, one day's work, one team's work, one span of a beam—or the entire structure?

This may seem a rather minor point, but consider the following: if a single-span beam is cast in one pour, by one team of workers, on one piece of formwork, then there is no problem. The probability of dead load forces exceeding the set limits to variation is equal to the value assumed by the authors of the code of practice.

However, for a long multi-span beam which is cast in a number of sections, it is not so clear. To look at an extreme example, consider a long cable-stayed bridge, for which the deck girders are suspended from a large number of cables and the deck is constructed from a large number of sections (Fig. 1). Such a bridge would be designed so that dead loads are carried almost entirely by axial forces in the cables and main girders, with minimal reliance on beam bending moments. If substantial deviations in dead load occurred in a systematic fashion along its length, large bending moments could be induced.

If the unit which was considered in assessing the probability of dead load variations was one span of the bridge, and load factor analysis was applied, the design loading for many sections would involve application of maximum load factor to dead load throughout the centre span and minimum for the side.
spans. This would generate large bending moments in the main girders and would prove an extremely onerous design condition. The IDR is even more severe in its requirements than this—it demands that, for each section considered, the differing load factors are applied to the appropriate influence line in such a way as to generate the worst possible effect, resulting in a different pattern of dead load variation being considered for every element of the bridge.

36. If, however, the unit considered was one segment of construction, then each span would have a total dead load very close to the theoretical value, because of the large number of units considered, and systematic variations of the kind described would be unlikely. As a result, bending moments induced would be slight.

37. In this case, the event considered in the probability analysis turns out to be crucial. It is hard to see any logic in defining the event on the basis of a span of the structure, as was first suggested, and it is still harder to believe that a segment of the influence line would be appropriate. (This is what one would have to do to make it consistent with IDR's recommended analysis.) Almost any of the alternatives suggested previously would seem to be more logical, although the true picture is likely to be a complex combination of all of them. Although the structure considered here might be thought to be an extreme case, consideration of other cases (e.g., an arch, or a continuous two-span beam cast on one set of formwork in one operation) shows that the analysis is of general application. It is also worth remembering that a claimed advantage of load factor codes is an ability to deal satisfactorily with complex structures.

38. Unless it can be demonstrated that dead load variations clearly associate with the curves of a structure's influence lines, or at the very least with spans of the structure, it must be concluded that the onerous loadings the load factor methods propose for some structures are quite unnecessary and misleading.

**Are the probabilities consistent?**

39. Clearly there are problems in defining local dead load variations in the proposed manner. However, let us assume for the moment that all the problems of defining the probability of dead load variation have been overcome and that the results can be applied to the design of structures. Assume that statistical analysis has successfully demonstrated that dead load varies between 1.0 and 1.4 times its characteristic value DL, with variations outside this range having a satisfactorily low probability of occurring. Let us assume that the probability of the load on a span reaching 1.4DL is 1/k.

40. The load factor analysis proposed in CP 110 can now be used to apply this information to some simple structures.
For a simply supported beam, the design loading is as shown in Fig. 2. The probability of this actually occurring clearly has the value just defined: $1/k$.

For a two-span beam, if one is designing the section at the centre support, the design loading is as shown in Fig. 3. If variations in both spans are independent and random, the probability of both reaching $1.4\text{DL}$ is $1/k^2$.

For a five-span beam, if the section at the centre of the middle span is being designed, the design loading is as shown in Fig. 4. The probability of spans reaching $1.4\text{DL}$ in the pattern shown is $1/k^3$. (If variations in dead load below $1.0\text{DL}$ were also considered, the probability would be $1/k^5$.)

If it is satisfactory to design a simply supported span against a probability of $1/k$, why must continuous beams be designed against probabilities of $1/k^2$, or even $1/k^3$? (Here engineering common sense would suggest that, if differing standards were being required, the more onerous one should apply to the simply supported beam, as its lack of scope for redistribution makes the consequences of failure more severe.)

For the simply supported case, the event shown is clearly the most severe associated with a probability of $1/k$. However, for the section in the five-span beam, one can consider alternative possible events with a probability of $1/k^3$. Returning to the original definition of dead load variations, it might be found that the probability of dead load on a span reaching (say) $2.0\text{DL}$ is $1/k^3$. Applying this to the centre span and $1.0\text{DL}$ to all other spans would give a loading which has a probability of $1/k^3$, but which is more severe than the loading specified.

Load factor analysis is proposed as a method whereby, for any structure, once the probabilities of various deviations from the expected loads have been satisfactorily defined, it will be possible to apply these in the design of a structure so that the probability of it reaching the ultimate limit state is a known, satisfactory value. If the analysis presented here is accepted as correct and relevant, the following important conclusions can be drawn.

First, even if loading variations were completely satisfactorily defined
in statistical and probability terms, application of the load factor method in this form would produce design loadings which had wildly different probabilities of occurrence for different structures and for different elements within the same structure.

45. Secondly, the loading defined is in many cases not the worst possible loading with its own probability of occurrence. Thus in these cases the loading proposed is not a satisfactory design loading.

46. Therefore the claim that the complexity of this load factor analysis can be justified because the results it produces are consistent and rational is shown to be false.

Conclusions

47. These arguments lead to the following conclusions.

(a) Despite the complexity of the arithmetic manipulations involved, the load factor methods give a rather unsatisfactory assessment of structural safety. The stress factor approach is superior in many respects.

(b) There are serious problems in attempting to define structural design completely in terms of classical probability theory.

(c) The complex treatment of dead load analysis adopted in CP 110, BS 5628 and the IDR has serious theoretical flaws which deny any claim of logic or consistency.

48. Unless either the propositions set out here are shown to be false or irrelevant, or else other justifications for the method are found, it is hard to escape the conclusion that the extra complexity of load factor analysis is not worthwhile and it should be discarded for structural design.

49. Although the theoretical claims made for the stress factor method have never been as grandiose as those made for load factor analysis, it can be shown that, provided there is always some scope for redistribution within a continuous structure, it will always produce structures with a reliable factor of safety. In fact, with regard to the problems investigated here, it seems to come through with fewer obvious weaknesses than the load factor method.

50. Points to consider if it is decided to revert to stress factor design are as follows.

51. Stress factor design is familiar and has been found satisfactory by generations of engineers. It is also a standard approach in various other branches of engineering.

52. Plastic design is still quite possible; the only large class of structures in the UK so far designed by plastic theory—steel portal frames—has generally been designed by the stress factor approach.

53. If the high effective factor of safety which the stress factor approach gives to structures and elements carrying mainly dead load is considered unnecessary, this can be easily dealt with by addition of a clause (§ 29). The idea introduced in the new masonry code of relating permissible stress to standard of construction can be dealt with similarly.

54. Current (load factor) codes can be easily adapted by reduction of permissible stresses by the appropriate factor, and use of the traditional 'permissible overstress' method to deal with exceptional loadings. Although older codes are generally satisfactory as they stand, there is always scope for improvements which take into account new ideas and knowledge (e.g., CP 114 could be revised
to incorporate CP 110's proposals on shear). Clauses to deal with plastic
design may be inserted where necessary.

55. Tabulated permissible stresses should incorporate all relevant safety
factors. (In BS 5400 and BS 5628, some of the factors to be applied are con-
stants and are specified in the codes, yet they are not incorporated into the
quoted stresses and must be applied by the engineer in every calculation. This
seems absurd.)

56. While statistical probability analysis is an extremely useful tool in the
estimation of suitable values for loads and material strengths, its limitations
must be borne in mind.

Addendum

57. Colleagues have pointed out that the new code for water-retaining
structures (BS 5337) also allows the use of the load factor approach. It is
fortunate that it also gives a stress factor method, as the load factor approach
seems completely wrong for this application. For water-retaining structures, the
ultimate limit state is quite unimportant—watertightness is assured by control
of strains and cracking. Also, as the principal imposed load, the weight of water,
is limited by the capacity of the tank, any load factor value greater than that
required to cover inaccuracies in analysis and construction has little meaning.

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